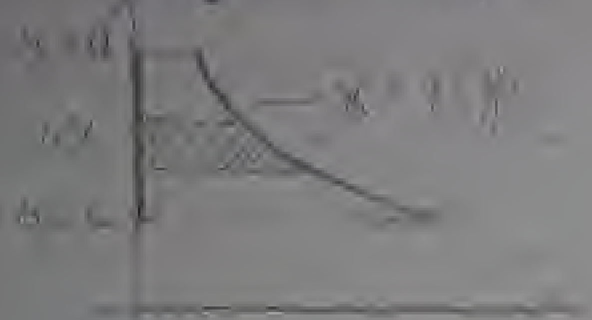
# Application of Integration

## Computing Areas in Cartesian Coordinate

**Horizontal Strip:**



$$A = \int_c^d f(y)\,dy$$

**Vertical Strip:**

**Example:-**

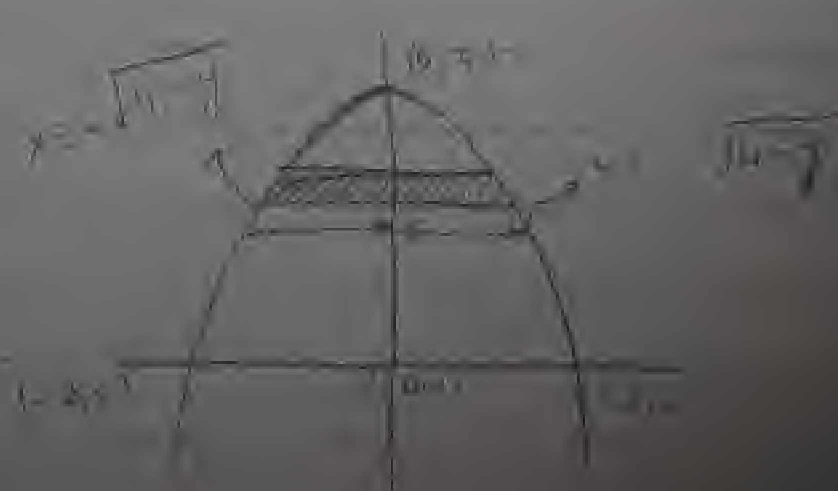
Find the area of the region bounded by the curve $y = 4 - x^2$ and the $x$-axis.

(i) By using vertical strips:

$$A = \int_{-2}^{2} (4 - x^2)\,dx = \frac{32}{3}$$

(ii) By using horizontal strips:

$$A = 2\int_0^4 \sqrt{4-y}\,dy = \frac{32}{3}$$



**Remark:** (sign of the area)

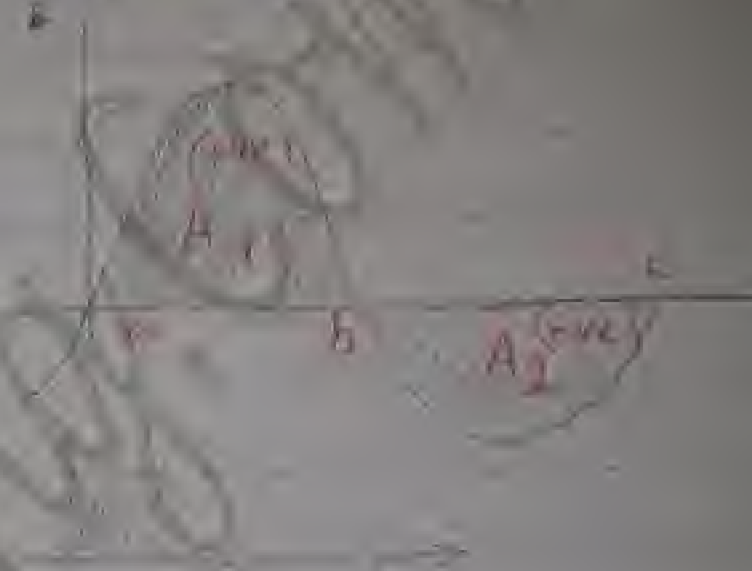
a) if $f(x) \geq 0$ over $[a, b] \Rightarrow A_1 = \int_a^b f(x)\,dx \geq 0$

b) if $f(x) \leq 0$ over $[b, c] \Rightarrow A_2 = \int_b^c f(x)\,dx \leq 0$

1) This implies that the total Area A bounded by $y = f(x)$ [illegible] $[a, c]$ [illegible]

$A = |A_1| + |A_2|$

2) [illegible]

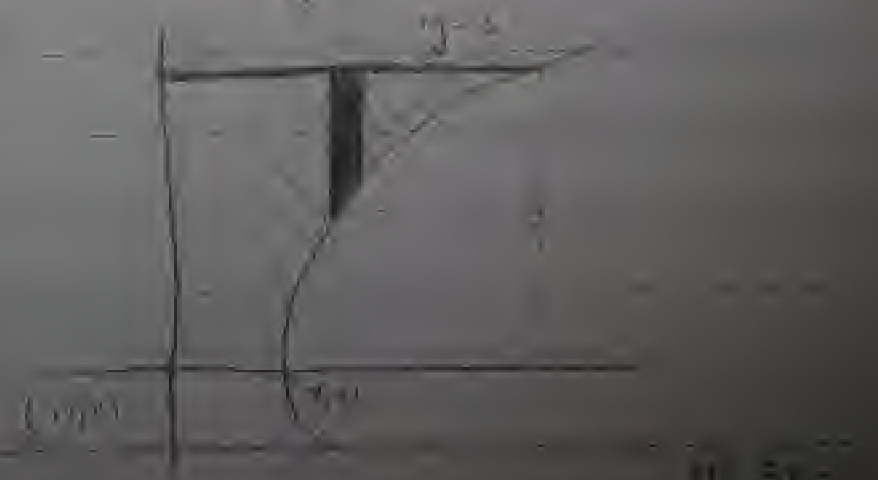



Find the area of the region bounded by the curve $y^2 = x - 1$, the y-axis, the x-axis and the line $y = 3$

[illegible]


Method 2

$A = \int_0 3\,dx + \int_1^{10} (3 - \sqrt{x-1})\,d$



## (6.1) Areas between Curves :-

### A1) Vertical Strips :-

$$A = \int_a^b [f(x) - g(x)]\, dx$$


### A2) Horizontal Strips :-

$$A_1 = \int [\ldots]\, dy$$

$$A_2 = \int [\ldots]\, dy$$

$$A = A_1 + A_2$$

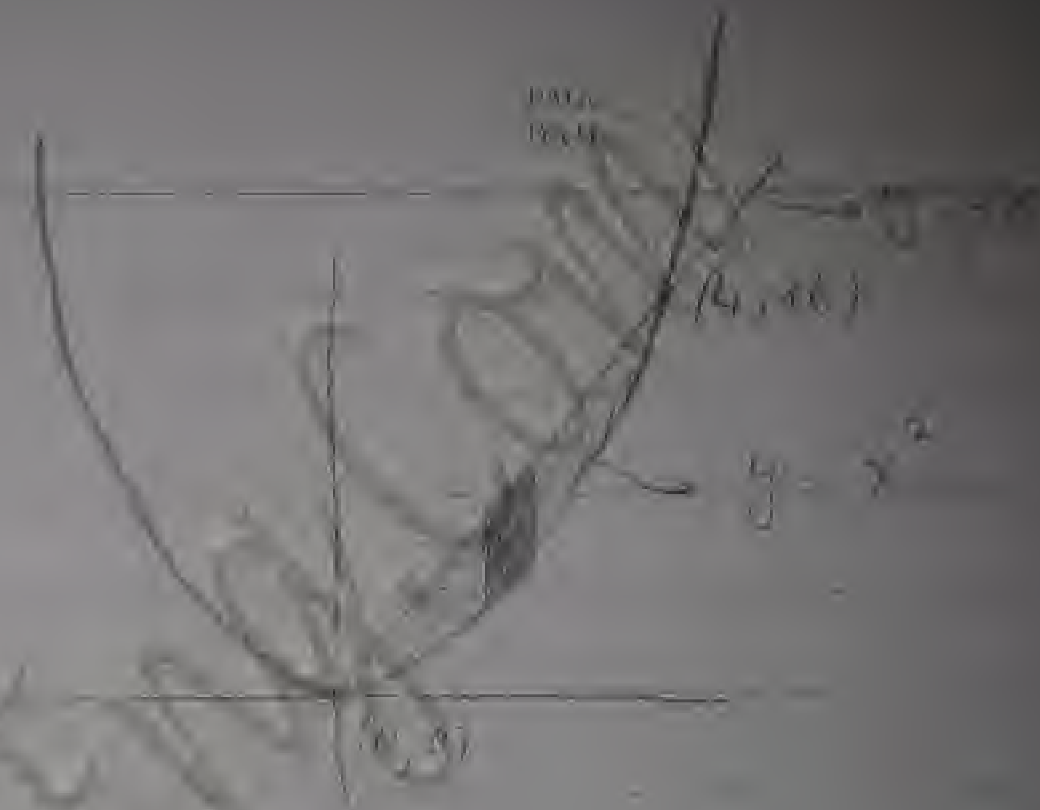
[illegible] intersection

## Guidelines for finding the Area :-

1) Sketch the given curve.
2) Sketch a typical vertical (horizontal) rectangle (strip)
3) Express the area in (1) and (2)
4) Consider the symmetry if any.
5) Area is always positive (+ve)

(1) Vertical Strips

$$A = \int_0^4 (4x - x^2)\,dx$$




(2) Horizontal Strips:-

$$A = \int_0^{16} \left(\sqrt{y} - \frac{y}{4}\right) dy$$

Find the area:-

$$A_1 = \int^{16} \sqrt{y} - x \, dy$$

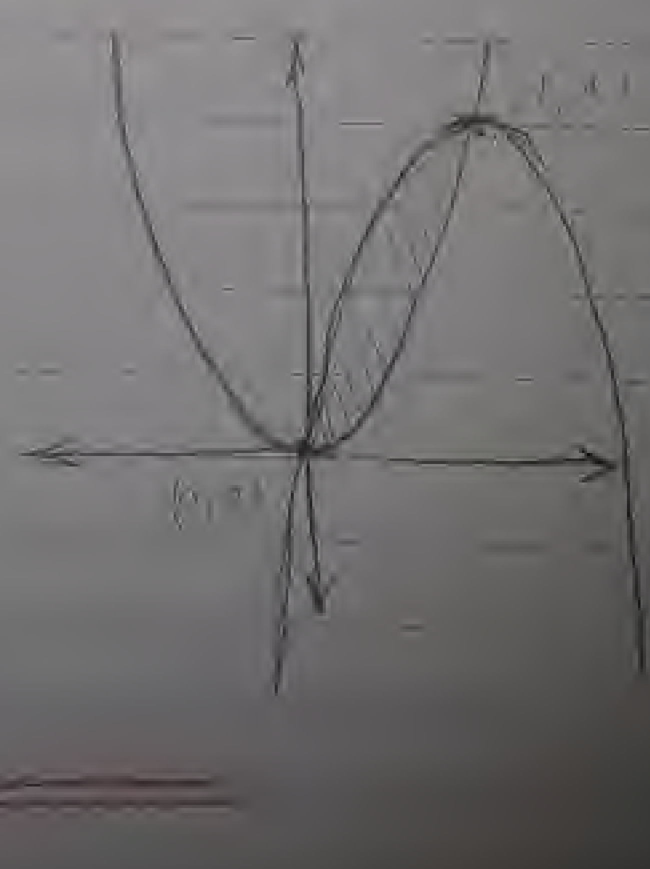
$$A_2 = \int_4^{} \frac{y}{4} - x \, dy$$

$$A = A_1 - A_2$$

Example :-

Find the area of the region bounded by the curve:-

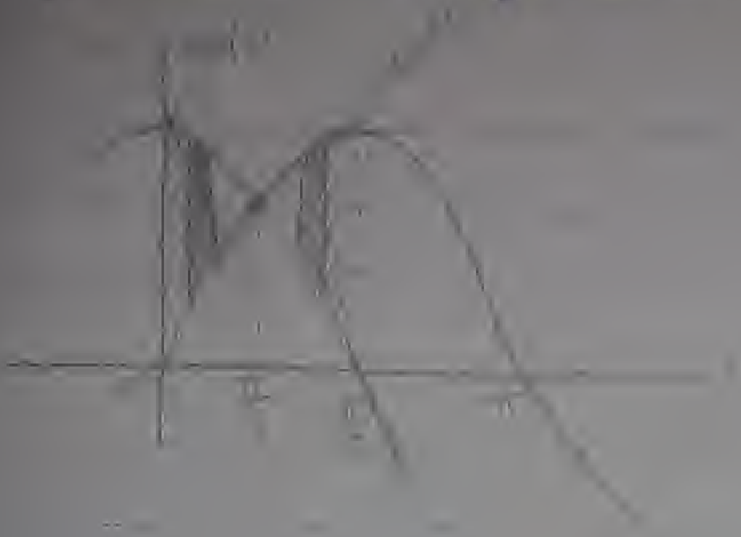
1) $y = x^2$ & $y = 2x - x^2$



$$A = \int_0^1 [(2x - x^2) - (x^2)] \, dx = \frac{1}{3}$$

4) $y = \sin(x)$ & $y = \cos(x)$

$x = 0 \to x = \frac{\pi}{2}$



$A_1 = \int [illegible]$

$A_2 = \int_{\pi}^{[illegible]} [illegible] \, dx$

3) $y = x - 1$ & $y^2 = 2x + 6$

$A = \int_{-2}^{[illegible]} [illegible] \, dy$

$A_2 = \int_{-1}^{5} \left[ \sqrt{2x+6} - (x-1) \right] dx$

$A_1 = 2 \int_{-3}^{-1} \left[ \sqrt{2x+6} - 0 \right] dx$